

# Solving linear and quadratic simultaneous equations

## A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

## Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

## Examples

**Example 1** Solve the simultaneous equations  $y = x + 1$  and  $x^2 + y^2 = 13$

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| $x^2 + (x + 1)^2 = 13$ $x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$ $2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$ <p>So <math>x = 2</math> or <math>x = -3</math></p> <p>Using <math>y = x + 1</math><br/>           When <math>x = 2</math>, <math>y = 2 + 1 = 3</math><br/>           When <math>x = -3</math>, <math>y = -3 + 1 = -2</math></p> <p>So the solutions are<br/> <math>x = 2, y = 3</math> and <math>x = -3, y = -2</math></p> <p>Check:</p> <p>equation 1: <math>3 = 2 + 1</math>            YES<br/>                         and <math>-2 = -3 + 1</math>        YES</p> <p>equation 2: <math>2^2 + 3^2 = 13</math>        YES<br/>                         and <math>(-3)^2 + (-2)^2 = 13</math> YES</p> | <ol style="list-style-type: none"> <li><b>1</b> Substitute <math>x + 1</math> for <math>y</math> into the second equation.</li> <li><b>2</b> Expand the brackets and simplify.</li> <li><b>3</b> Factorise the quadratic equation.</li> <li><b>4</b> Work out the values of <math>x</math>.</li> <li><b>5</b> To find the value of <math>y</math>, substitute both values of <math>x</math> into one of the original equations.</li> <li><b>6</b> Substitute both pairs of values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol> |
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**Example 2** Solve  $2x + 3y = 5$  and  $2y^2 + xy = 12$  simultaneously.

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| $x = \frac{5-3y}{2}$ $2y^2 + \left(\frac{5-3y}{2}\right)y = 12$ $2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$ $(y+8)(y-3) = 0$ <p>So <math>y = -8</math> or <math>y = 3</math></p> <p>Using <math>2x + 3y = 5</math><br/>           When <math>y = -8</math>, <math>2x + 3 \times (-8) = 5</math>, <math>x = 14.5</math><br/>           When <math>y = 3</math>, <math>2x + 3 \times 3 = 5</math>, <math>x = -2</math></p> <p>So the solutions are<br/> <math>x = 14.5</math>, <math>y = -8</math> and <math>x = -2</math>, <math>y = 3</math></p> <p>Check:<br/>           equation 1: <math>2 \times 14.5 + 3 \times (-8) = 5</math> YES<br/>                             and <math>2 \times (-2) + 3 \times 3 = 5</math> YES<br/>           equation 2: <math>2 \times (-8)^2 + 14.5 \times (-8) = 12</math> YES<br/>                             and <math>2 \times (3)^2 + (-2) \times 3 = 12</math> YES</p> | <ol style="list-style-type: none"> <li>1 Rearrange the first equation.</li> <li>2 Substitute <math>\frac{5-3y}{2}</math> for <math>x</math> into the second equation. Notice how it is easier to substitute for <math>x</math> than for <math>y</math>.</li> <li>3 Expand the brackets and simplify.</li> <li>4 Factorise the quadratic equation.</li> <li>5 Work out the values of <math>y</math>.</li> <li>6 To find the value of <math>x</math>, substitute both values of <math>y</math> into one of the original equations.</li> <li>7 Substitute both pairs of values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol> |
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## Practice

Solve these simultaneous equations.

1  $y = 2x + 1$   
 $x^2 + y^2 = 10$

2  $y = 6 - x$   
 $x^2 + y^2 = 20$

3  $y = x - 3$   
 $x^2 + y^2 = 5$

4  $y = 9 - 2x$   
 $x^2 + y^2 = 17$

5  $y = 3x - 5$   
 $y = x^2 - 2x + 1$

6  $y = x - 5$   
 $y = x^2 - 5x - 12$

7  $y = x + 5$   
 $x^2 + y^2 = 25$

8  $y = 2x - 1$   
 $x^2 + xy = 24$

9  $y = 2x$   
 $y^2 - xy = 8$

10  $2x + y = 11$   
 $xy = 15$

## Extend

11  $x - y = 1$   
 $x^2 + y^2 = 3$

12  $y - x = 2$   
 $x^2 + xy = 3$

## Answers

**1**  $x = 1, y = 3$

$$x = -\frac{9}{5}, y = -\frac{13}{5}$$

**2**  $x = 2, y = 4$

$$x = 4, y = 2$$

**3**  $x = 1, y = -2$

$$x = 2, y = -1$$

**4**  $x = 4, y = 1$

$$x = \frac{16}{5}, y = \frac{13}{5}$$

**5**  $x = 3, y = 4$

$$x = 2, y = 1$$

**6**  $x = 7, y = 2$

$$x = -1, y = -6$$

**7**  $x = 0, y = 5$

$$x = -5, y = 0$$

**8**  $x = -\frac{8}{3}, y = -\frac{19}{3}$

$$x = 3, y = 5$$

**9**  $x = -2, y = -4$

$$x = 2, y = 4$$

**10**  $x = \frac{5}{2}, y = 6$

$$x = 3, y = 5$$

**11**  $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$$

**12**  $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$$