Solving simultaneous equations graphically
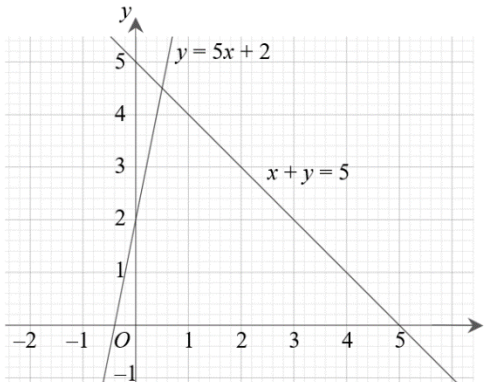
A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.
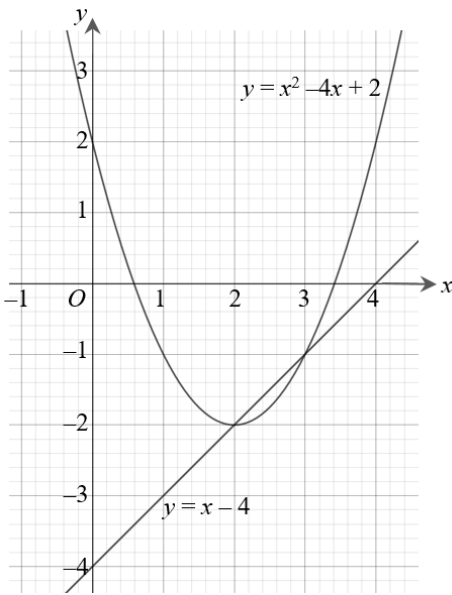
Examples

Example 1   Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

$y = 5 - x$	**1** Rearrange the equation $x + y = 5$ to make y the subject.
$y = 5 - x$ has gradient -1 and y-intercept 5. $y = 5x + 2$ has gradient 5 and y-intercept 2. 	**2** Plot both graphs on the same grid using the gradients and y-intercepts.

Lines intersect at
    $x = 0.5, y = 4.5$
 	
 Check:
 First equation $y = 5x + 2$:
    $4.5 = 5 \times 0.5 + 2$ YES
 Second equation $x + y = 5$:
    $0.5 + 4.5 = 5$ YES
	3 The solutions of the simultaneous equations are the point of intersection.
	
	 4 Check your solutions by substituting the values into both equations.

Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	−1	−2	−1	2

The line and curve intersect at
$x = 3, y = -1$ and $x = 2, y = -2$

Check:
First equation $y = x - 4$:
$\quad -1 = 3 - 4 \qquad\qquad$ YES
$\quad -2 = 2 - 4 \qquad\qquad$ YES
Second equation $y = x^2 - 4x + 2$:
$\quad -1 = 3^2 - 4 \times 3 + 2 \qquad$ YES
$\quad -2 = 2^2 - 4 \times 2 + 2 \qquad$ YES

1 Construct a table of values and calculate the points for the quadratic equation.

2 Plot the graph.

3 Plot the linear graph on the same grid using the gradient and y-intercept.
$y = x - 4$ has gradient 1 and y-intercept −4.

4 The solutions of the simultaneous equations are the points of intersection.

5 Check your solutions by substituting the values into both equations.

Practice

1 Solve these pairs of simultaneous equations graphically.
   a $\quad y = 3x - 1$ and $y = x + 3$
   b $\quad y = x - 5$ and $y = 7 - 5x$
   c $\quad y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.
   a $\quad x + y = 0$ and $y = 2x + 6$
   b $\quad 4x + 2y = 3$ and $y = 3x - 1$
   c $\quad 2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

  a $y = x - 1$ and $y = x^2 - 4x + 3$

  b $y = 1 - 3x$ and $y = x^2 - 3x - 3$

  c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 **a** Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

    i graphically

    ii algebraically to 2 decimal places.

  b Which method gives the more accurate solutions? Explain your answer.

Answers

1 a $x = 2, y = 5$
   b $x = 2, y = -3$
   c $x = -0.5, y = 2.5$

2 a $x = -2, y = 2$
   b $x = 0.5, y = 0.5$
   c $x = -1, y = -2$

3 a $x = 1, y = 0$ and $x = 4, y = 3$
   b $x = -2, y = 7$ and $x = 2, y = -5$
   c $x = -2, y = 5$ and $x = -1, y = 4$

4 $x = -3, y = 4$ and $x = 4, y = -3$

5 a i $x = 2.5, y = -2$ and $x = -0.5, y = 4$
      ii $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$
   b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.