Completing the square

> **A LEVEL LINKS**
> **Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1   Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$	**1** Write $x^2 + bx + c$ in the form
$= (x + 3)^2 - 9 - 2$	$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$
$= (x + 3)^2 - 11$	**2** Simplify

Example 2   Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $$a\left(x^2 + \frac{b}{a}x\right) + c$$
$= 2\left(x^2 - \frac{5}{2}x\right) + 1$	**2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$
$= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$	
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	**4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

    a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

    c $x^2 - 8x$ **d** $x^2 + 6x$

    e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

    a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

    c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

    a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

    c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 a $(x+2)^2 - 1$ **b** $(x-5)^2 - 28$

c $(x-4)^2 - 16$ **d** $(x+3)^2 - 9$

e $(x-1)^2 + 6$ **f** $\left(x+\dfrac{3}{2}\right)^2 - \dfrac{17}{4}$

2 a $2(x-2)^2 - 24$ **b** $4(x-1)^2 - 20$

c $3(x+2)^2 - 21$ **d** $2\left(x+\dfrac{3}{2}\right)^2 - \dfrac{25}{2}$

3 a $2\left(x+\dfrac{3}{4}\right)^2 + \dfrac{39}{8}$ **b** $3\left(x-\dfrac{1}{3}\right)^2 - \dfrac{1}{3}$

c $5\left(x+\dfrac{3}{10}\right)^2 - \dfrac{9}{20}$ **d** $3\left(x+\dfrac{5}{6}\right)^2 + \dfrac{11}{12}$

4 $(5x+3)^2 + 3$