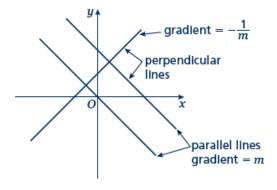
# Parallel and perpendicular lines

#### A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

# **Key points**

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation y = mx + c has gradient  $-\frac{1}{m}$ .



# **Examples**

**Example 1** Find the equation of the line parallel to y = 2x + 4 which passes through the point (4, 9).

y = 2x + 4 $m = 2$	1 As the lines are parallel they have the same gradient.
y = 2x + c	2 Substitute $m = 2$ into the equation of a straight line $y = mx + c$ .
$9 = 2 \times 4 + c$	3 Substitute the coordinates into the equation $y = 2x + c$
9 = 8 + c	4 Simplify and solve the equation.
c=1	
y = 2x + 1	5 Substitute $c = 1$ into the equation
	y = 2x + c

**Example 2** Find the equation of the line perpendicular to y = 2x - 3 which passes through the point (-2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	1 As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$ .
$y = -\frac{1}{2}x + c$	2 Substitute $m = -\frac{1}{2}$ into $y = mx + c$ .
$5 = -\frac{1}{2} \times (-2) + c$	3 Substitute the coordinates (-2, 5) into the equation $y = -\frac{1}{2}x + c$
5 = 1 + c $c = 4$	4 Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	5 Substitute $c = 4$ into $y = -\frac{1}{2}x + c$ .

Example 3 A line passes through the points (0, 5) and (9, -1). Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$$x_1 = 0$$
,  $x_2 = 9$ ,  $y_1 = 5$  and  $y_2 = -1$ 

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$$

$$= \frac{-6}{9} = -\frac{2}{3}$$
1 3

$$-\frac{1}{m} = \frac{3}{2}$$

$$y = \frac{3}{2}x + c$$

Midpoint = 
$$\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$$

$$2 = \frac{3}{2} \times \frac{9}{2} + \epsilon$$

$$c = -\frac{19}{4}$$

$$y = \frac{3}{2}x - \frac{19}{4}$$

- 1 Substitute the coordinates into the equation  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to work out the gradient of the line.
- 2 As the lines are perpendicular, the gradient of the perpendicular line
- 3 Substitute the gradient into the equation y = mx + c.
- Work out the coordinates of the midpoint of the line.
- 5 Substitute the coordinates of the midpoint into the equation.
- **6** Simplify and solve the equation.
- 7 Substitute  $c = -\frac{19}{4}$  into the equation

$$y = \frac{3}{2}x + c.$$

# **Practice**

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

**a** 
$$y = 3x + 1$$
 (3, 2)

**h** 
$$y = 3 - 2x$$
 (1.3)

**a** 
$$y = 3x + 1$$
 (3, 2)   
**b**  $y = 3 - 2x$  (1, 3)   
**c**  $2x + 4y + 3 = 0$  (6, -3)   
**d**  $2y - 3x + 2 = 0$  (8, 20)

**d** 
$$2y-3x+2=0$$
 (8, 20)

Find the equation of the line perpendicular to  $y = \frac{1}{2}x - 3$  which 2 passes through the point (-5, 3).

If  $m = \frac{a}{b}$  then the negative reciprocal  $-\frac{1}{m} = -\frac{b}{a}$ 

Find the equation of the line perpendicular to each of the given lines and which passes through 3 each of the given points.

**a** 
$$y = 2x - 6$$
 (4, 0)

**b** 
$$y = -\frac{1}{3}x + \frac{1}{2}$$
 (2, 13)

$$\mathbf{c}$$
  $x-4y-4=0$  (5, 15)

**d** 
$$5y + 2x - 5 = 0$$
  $(6, 7)$ 



4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

$$a$$
 (4, 3), (-2, -9)

### **Extend**

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

$$\mathbf{a} \qquad y = 2x + 3$$
$$y = 2x - 7$$

$$y = 3x 
 2x + y - 3 = 0$$

$$\mathbf{c} \qquad y = 4x - 3 \\
4y + x = 2$$

$$\mathbf{d} \quad 3x - y + 5 = 0$$
$$x + 3y = 1$$

**e** 
$$2x + 5y - 1 = 0$$
 **f**  $y = 2x + 7$ 

$$\mathbf{f} \qquad 2x - y = 6$$
$$6x - 3y + 3 = 0$$

- 6 The straight line  $L_1$  passes through the points A and B with coordinates (-4, 4) and (2, 1), respectively.
  - a Find the equation of  $L_1$  in the form ax + by + c = 0

The line  $L_2$  is parallel to the line  $L_1$  and passes through the point C with coordinates (-8, 3).

**b** Find the equation of  $\mathbf{L}_2$  in the form ax + by + c = 0

The line  $L_3$  is perpendicular to the line  $L_1$  and passes through the origin.

c Find an equation of  $L_3$ 

## **Answers**

1 **a** 
$$y = 3x - 7$$

$$y = -2x + 5$$

**c** 
$$y = -\frac{1}{2}x$$

1 **a** 
$$y = 3x - 7$$
 **b**  $y = -2x + 5$  **c**  $y = -\frac{1}{2}x$  **d**  $y = \frac{3}{2}x + 8$ 

2 
$$y = -2x - 7$$

**3 a** 
$$y = -\frac{1}{2}x + 2$$
 **b**  $y = 3x + 7$ 

**b** 
$$y = 3x + 7$$

**c** 
$$y = -4x + 35$$

**c** 
$$y = -4x + 35$$
 **d**  $y = \frac{5}{2}x - 8$ 

**4 a** 
$$y = -\frac{1}{2}x$$

$$\mathbf{b} \qquad y = 2x$$

Neither b

Perpendicular c

**d** Perpendicular

e Neither

f Parallel

**6 a** 
$$x + 2y - 4 = 0$$
 **b**  $x + 2y + 2 = 0$  **c**  $y = 2x$ 

**b** 
$$x + 2y + 2 = 0$$

$$\mathbf{c}$$
  $y = 2x$