

# Translating graphs

## A LEVEL LINKS

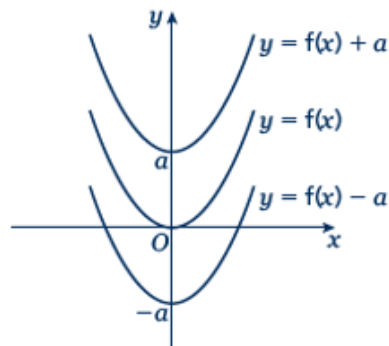
**Scheme of work:** 1f. Transformations – transforming graphs –  $f(x)$  notation

### Key points

- The transformation  $y = f(x) \pm a$  is a translation of  $y = f(x)$  parallel to the  $y$ -axis; it is a vertical translation.

As shown on the graph,

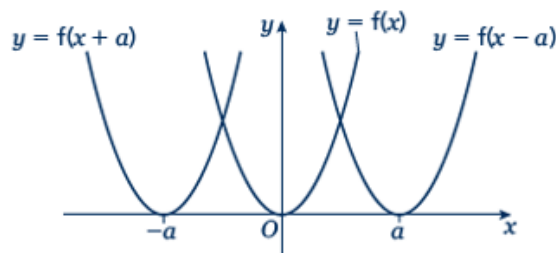
- $y = f(x) + a$  translates  $y = f(x)$  up
- $y = f(x) - a$  translates  $y = f(x)$  down.



- The transformation  $y = f(x \pm a)$  is a translation of  $y = f(x)$  parallel to the  $x$ -axis; it is a horizontal translation.

As shown on the graph,

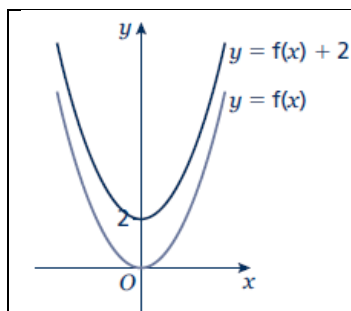
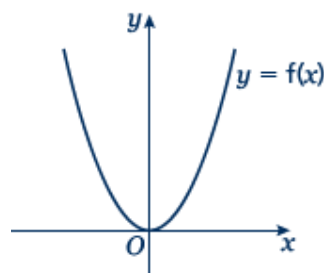
- $y = f(x + a)$  translates  $y = f(x)$  to the left
- $y = f(x - a)$  translates  $y = f(x)$  to the right.



### Examples

**Example 1** The graph shows the function  $y = f(x)$ .

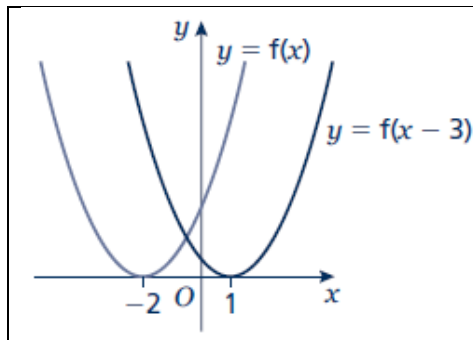
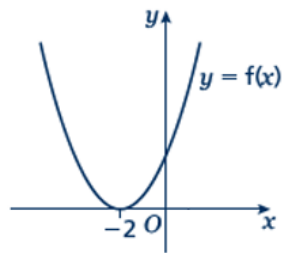
Sketch the graph of  $y = f(x) + 2$ .



For the function  $y = f(x) + 2$  translate the function  $y = f(x)$  2 units up.

**Example 2** The graph shows the function  $y = f(x)$ .

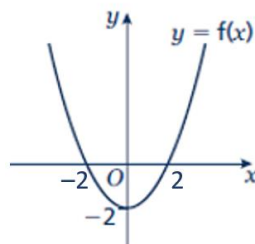
Sketch the graph of  $y = f(x - 3)$ .



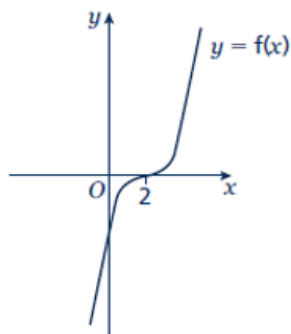
For the function  $y = f(x - 3)$  translate the function  $y = f(x)$  3 units right.

## Practice

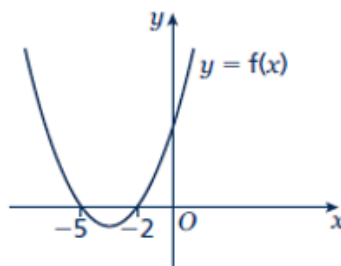
- 1** The graph shows the function  $y = f(x)$ .  
Copy the graph and on the same axes sketch and label the graphs of  $y = f(x) + 4$  and  $y = f(x + 2)$ .



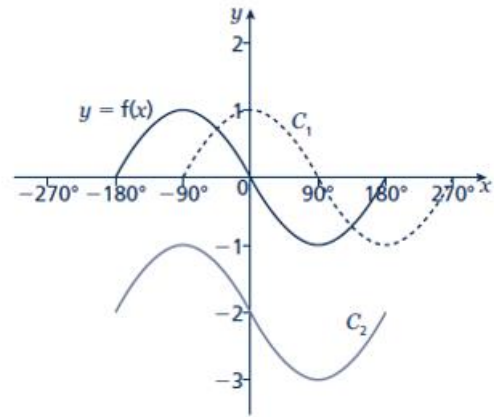
- 2** The graph shows the function  $y = f(x)$ .  
Copy the graph and on the same axes sketch and label the graphs of  $y = f(x + 3)$  and  $y = f(x) - 3$ .



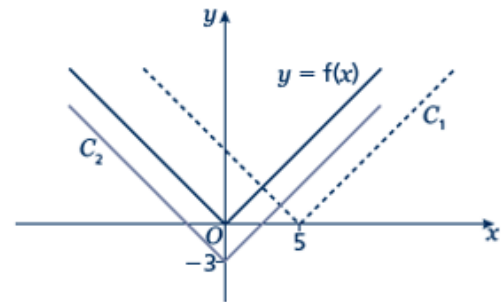
- 3** The graph shows the function  $y = f(x)$ .  
Copy the graph and on the same axes sketch the graph of  $y = f(x - 5)$ .



- 4 The graph shows the function  $y = f(x)$  and two transformations of  $y = f(x)$ , labelled  $C_1$  and  $C_2$ . Write down the equations of the translated curves  $C_1$  and  $C_2$  in function form.

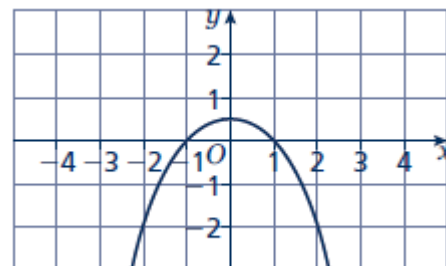


- 5 The graph shows the function  $y = f(x)$  and two transformations of  $y = f(x)$ , labelled  $C_1$  and  $C_2$ . Write down the equations of the translated curves  $C_1$  and  $C_2$  in function form.



- 6 The graph shows the function  $y = f(x)$ .

- a Sketch the graph of  $y = f(x) + 2$
- b Sketch the graph of  $y = f(x + 2)$



# Stretching graphs

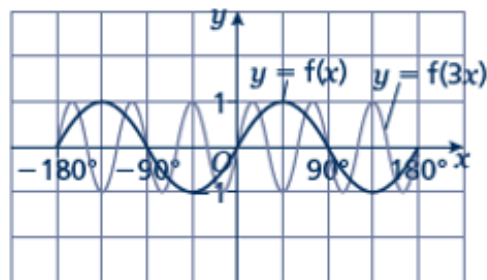
## A LEVEL LINKS

**Scheme of work:** 1f. Transformations – transforming graphs –  $f(x)$  notation

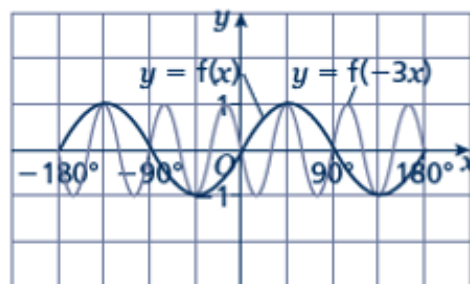
**Textbook:** Pure Year 1, 4.6 Stretching graphs

## Key points

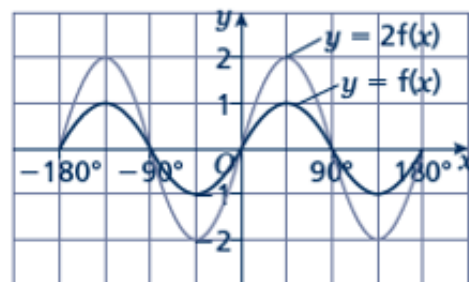
- The transformation  $y = f(ax)$  is a horizontal stretch of  $y = f(x)$  with scale factor  $\frac{1}{a}$  parallel to the  $x$ -axis.



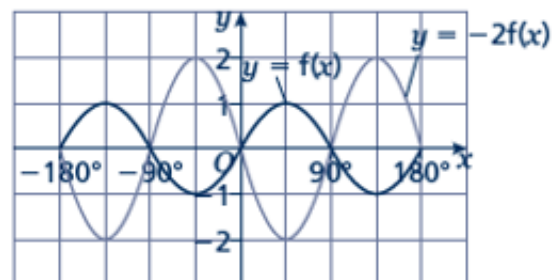
- The transformation  $y = f(-ax)$  is a horizontal stretch of  $y = f(x)$  with scale factor  $\frac{1}{a}$  parallel to the  $x$ -axis and then a reflection in the  $y$ -axis.



- The transformation  $y = af(x)$  is a vertical stretch of  $y = f(x)$  with scale factor  $a$  parallel to the  $y$ -axis.



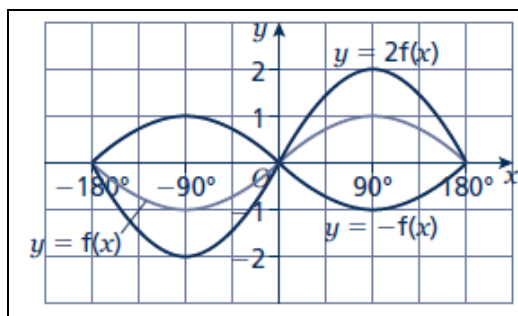
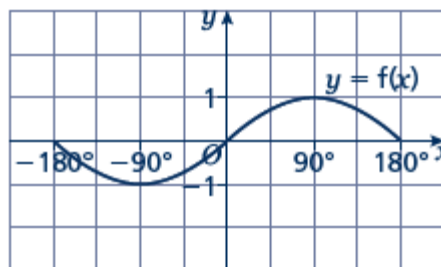
- The transformation  $y = -af(x)$  is a vertical stretch of  $y = f(x)$  with scale factor  $a$  parallel to the  $y$ -axis and then a reflection in the  $x$ -axis.



## Examples

**Example 3** The graph shows the function  $y = f(x)$ .

Sketch and label the graphs of  $y = 2f(x)$  and  $y = -f(x)$ .

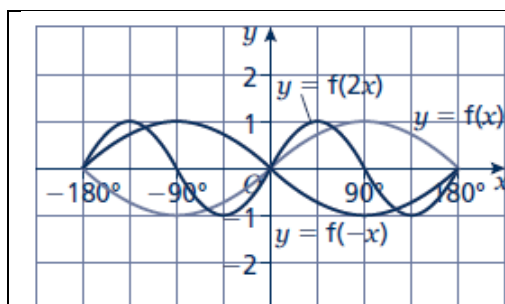
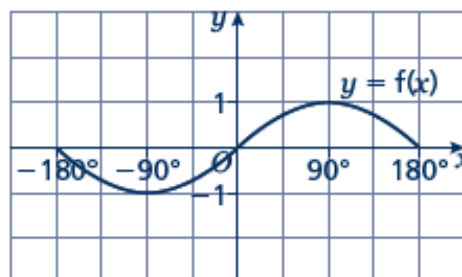


The function  $y = 2f(x)$  is a vertical stretch of  $y = f(x)$  with scale factor 2 parallel to the  $y$ -axis.

The function  $y = -f(x)$  is a reflection of  $y = f(x)$  in the  $x$ -axis.

**Example 4** The graph shows the function  $y = f(x)$ .

Sketch and label the graphs of  $y = f(2x)$  and  $y = f(-x)$ .

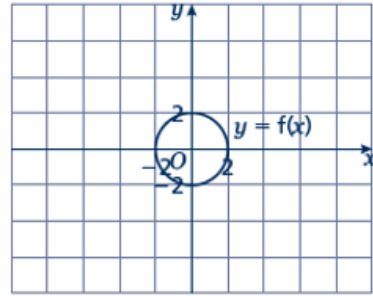


The function  $y = f(2x)$  is a horizontal stretch of  $y = f(x)$  with scale factor  $\frac{1}{2}$  parallel to the  $x$ -axis.

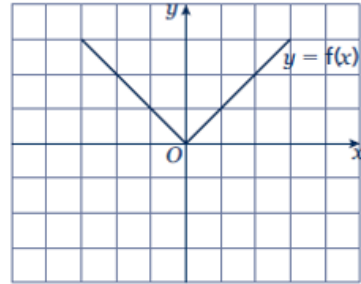
The function  $y = f(-x)$  is a reflection of  $y = f(x)$  in the  $y$ -axis.

## Practice

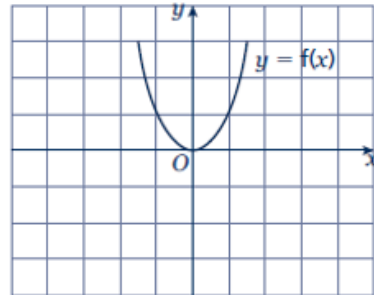
- 7 The graph shows the function  $y = f(x)$ .
- Copy the graph and on the same axes sketch and label the graph of  $y = 3f(x)$ .
  - Make another copy of the graph and on the same axes sketch and label the graph of  $y = f(2x)$ .



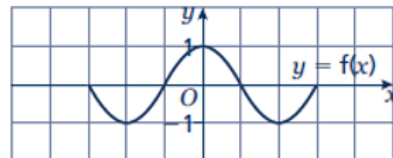
- 8 The graph shows the function  $y = f(x)$ . Copy the graph and on the same axes sketch and label the graphs of  $y = -2f(x)$  and  $y = f(3x)$ .



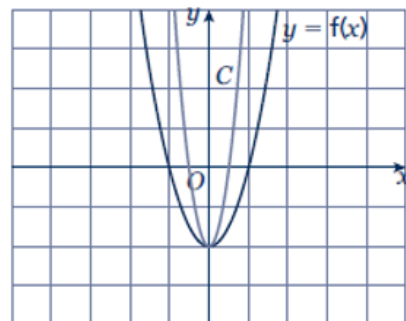
- 9 The graph shows the function  $y = f(x)$ . Copy the graph and, on the same axes, sketch and label the graphs of  $y = -f(x)$  and  $y = f(\frac{1}{2}x)$ .



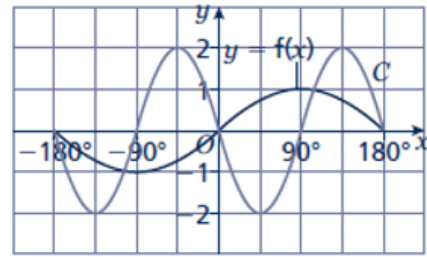
- 10 The graph shows the function  $y = f(x)$ . Copy the graph and, on the same axes, sketch the graph of  $y = -f(2x)$ .



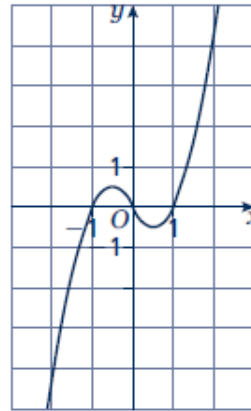
- 11 The graph shows the function  $y = f(x)$  and a transformation, labelled  $C$ . Write down the equation of the translated curve  $C$  in function form.



- 12 The graph shows the function  $y = f(x)$  and a transformation labelled  $C$ . Write down the equation of the translated curve  $C$  in function form.



- 13 The graph shows the function  $y = f(x)$ .
- Sketch the graph of  $y = -f(x)$ .
  - Sketch the graph of  $y = 2f(x)$ .

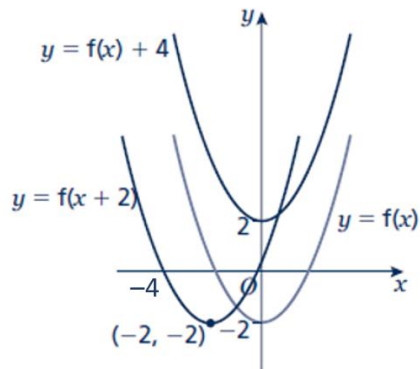


## Extend

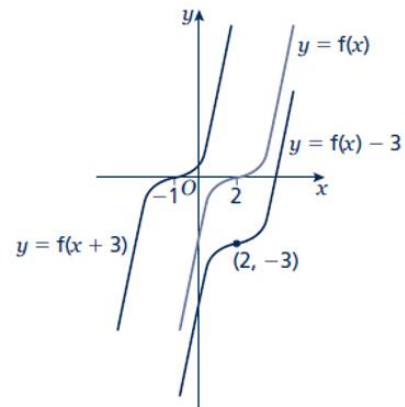
- 14
- Sketch and label the graph of  $y = f(x)$ , where  $f(x) = (x - 1)(x + 1)$ .
  - On the same axes, sketch and label the graphs of  $y = f(x) - 2$  and  $y = f(x + 2)$ .
- 15
- Sketch and label the graph of  $y = f(x)$ , where  $f(x) = -(x + 1)(x - 2)$ .
  - On the same axes, sketch and label the graph of  $y = f\left(-\frac{1}{2}x\right)$ .

## Answers

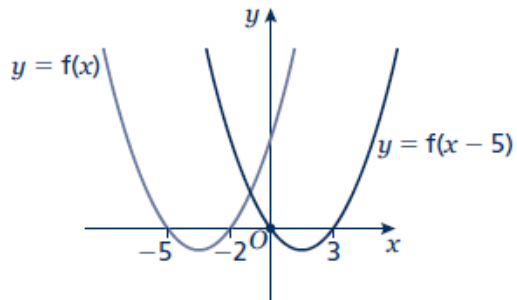
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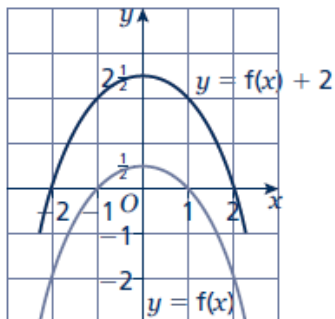
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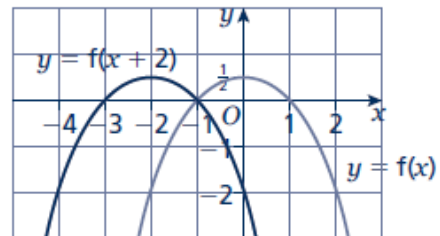
4  $C_1: y = f(x - 90^\circ)$   
 $C_2: y = f(x) - 2$

5  $C_1: y = f(x - 5)$   
 $C_2: y = f(x) - 3$

6 a

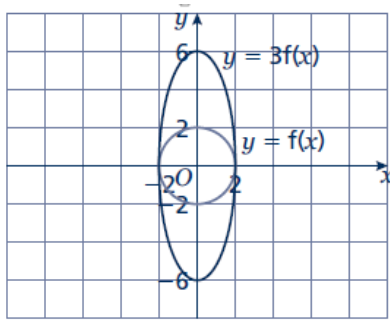


b

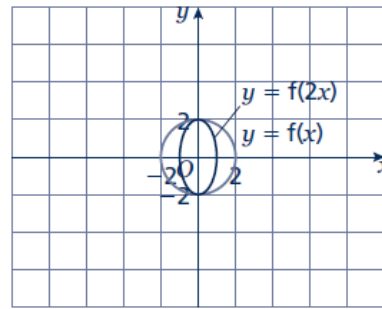




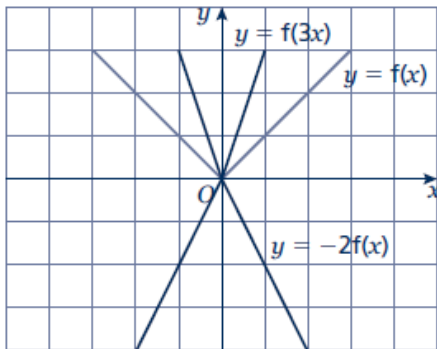
7 a



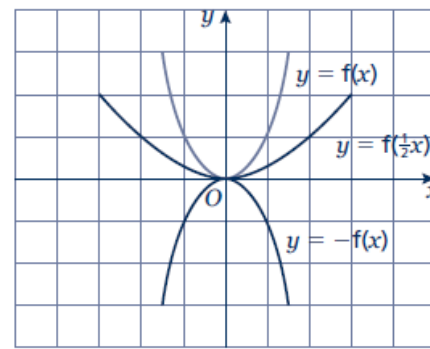
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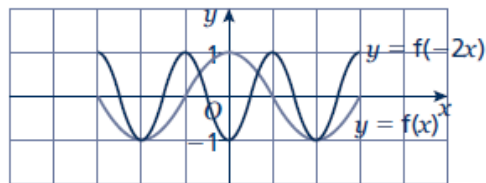
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9



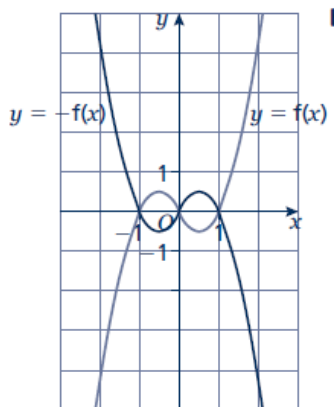
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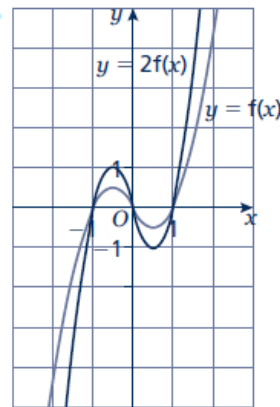
11  $y = f(2x)$

12  $y = -2f(2x)$  or  $y = 2f(-2x)$

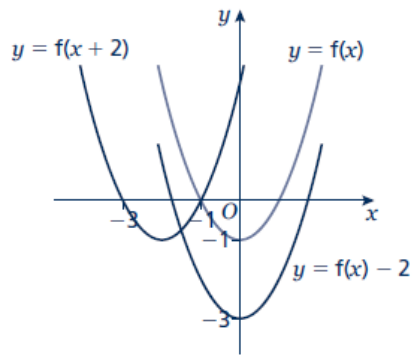
13 a



b



14



15

