

Rules of indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

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|------------|-----------------------------------------------------|
| $10^0 = 1$ | Any value raised to the power of zero is equal to 1 |
|------------|-----------------------------------------------------|

Example 2 Evaluate $9^{\frac{1}{2}}$

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|------------------------------------|----------------------------------------------|
| $9^{\frac{1}{2}} = \sqrt{9}$ $= 3$ | Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ |
|------------------------------------|----------------------------------------------|

Example 3 Evaluate $27^{\frac{2}{3}}$

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|-----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$ | <ol style="list-style-type: none"> 1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ 2 Use $\sqrt[3]{27} = 3$ |
|-----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|

Example 4 Evaluate 4^{-2}

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|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| $4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$ | <ol style="list-style-type: none"> 1 Use the rule $a^{-m} = \frac{1}{a^m}$ 2 Use $4^2 = 16$ |
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Example 5 Simplify $\frac{6x^5}{2x^2}$

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|----------------------------|----------------------------------------------------------------------------------------------------------|
| $\frac{6x^5}{2x^2} = 3x^3$ | $6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$ |
|----------------------------|----------------------------------------------------------------------------------------------------------|

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

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| $\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$ | <ol style="list-style-type: none"> 1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$ |
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Example 7 Write $\frac{1}{3x}$ as a single power of x

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|------------------------------------|---------------------------------------------------------------------------------------------------|
| $\frac{1}{3x} = \frac{1}{3}x^{-1}$ | Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged |
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Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

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|------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$ | <ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$ |
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Practice

1 Evaluate.

a 14^0

b 3^0

c 5^0

d x^0

2 Evaluate.

a $49^{\frac{1}{2}}$

b $64^{\frac{1}{3}}$

c $125^{\frac{1}{3}}$

d $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$

b $8^{\frac{5}{3}}$

c $49^{\frac{3}{2}}$

d $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2}

b 4^{-3}

c 2^{-5}

d 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$

b $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

d $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{(2x^2)^3}{4x^0}$

h $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$

b $27^{-\frac{2}{3}}$

c $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$

e $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

f $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x .

a $\frac{1}{x}$

b $\frac{1}{x^7}$

c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt[3]{x}}$

f $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3}

b x^0

c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{2}}$

f $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n .

a $5\sqrt{x}$

b $\frac{2}{x^3}$

c $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$

e $\frac{4}{\sqrt[3]{x}}$

f 3

Extend

10 Write as sums of powers of x .

a $\frac{x^5 + 1}{x^2}$

b $x^2\left(x + \frac{1}{x}\right)$

c $x^{-4}\left(x^2 + \frac{1}{x^3}\right)$

Answers

1 a 1

b 1

c 1

d 1

2 a 7

b 4

c 5

d 2

3 a 125

b 32

c 343

d 8

4 a $\frac{1}{25}$

b $\frac{1}{64}$

c $\frac{1}{32}$

d $\frac{1}{36}$

5 a $\frac{3x^3}{2}$

b $5x^2$

c $3x$

d $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$

f c^{-3}

g $2x^6$

h x

6 a $\frac{1}{2}$

b $\frac{1}{9}$

c $\frac{8}{3}$

d $\frac{1}{4}$

e $\frac{4}{3}$

f $\frac{16}{9}$

7 a x^{-1}

b x^{-7}

c $x^{\frac{1}{4}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{3}}$

f $x^{\frac{2}{3}}$

8 a $\frac{1}{x^3}$

b 1

c $\sqrt[5]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt{x}}$

f $\frac{1}{\sqrt[4]{x^3}}$

9 a $5x^{\frac{1}{2}}$

b $2x^{-3}$

c $\frac{1}{3}x^{-4}$

d $2x^{-\frac{1}{2}}$

e $4x^{-\frac{1}{3}}$

f $3x^0$

10 a $x^3 + x^{-2}$

b $x^3 + x$

c $x^{-2} + x^{-7}$